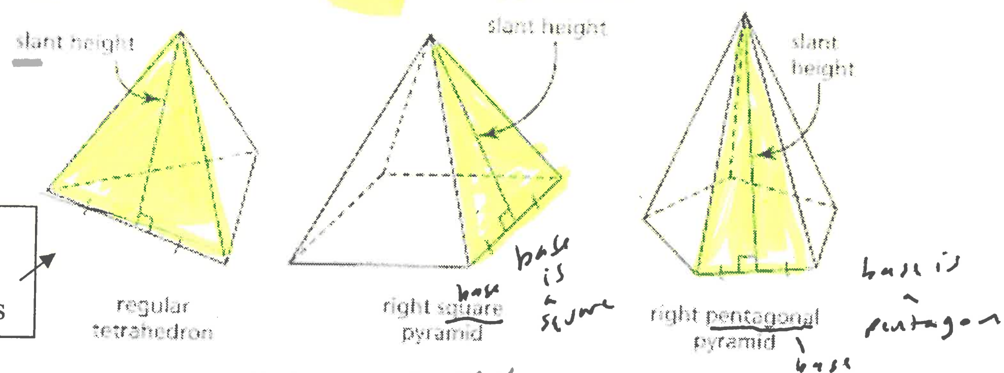
1.4 – Surface Areas of Right Pyramids and Right Cones p. 26

Surface area is measured in units squared .

Right Pyramids

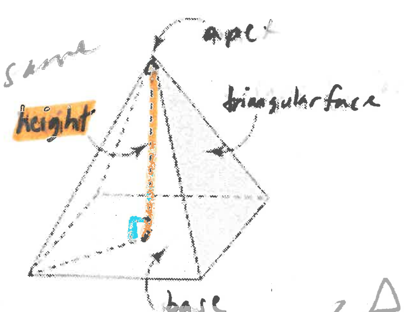
- The Surface area is the total area on the surface of an object.
- A right pyramid is a 3 – dimensional shape that has triangular faces and a base that is a polygon. The apex of the shape is directly above the centre of the base.
- A right **regular** pyramid has a base that is a **regular polygon** (all sides equal), which makes all the lateral faces the same.
- The shape **of the polygon** determines the name **of the pyramid**.

Tetrahedron:
base and sides are **equilateral** triangles

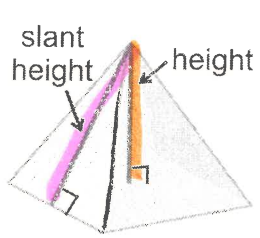


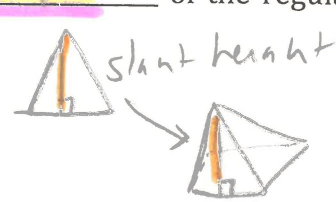
- The triangular faces meet at a point called the apex.
- The height of the pyramid is the perpendicular distance from the apex to the centre of the base.

all sides the same

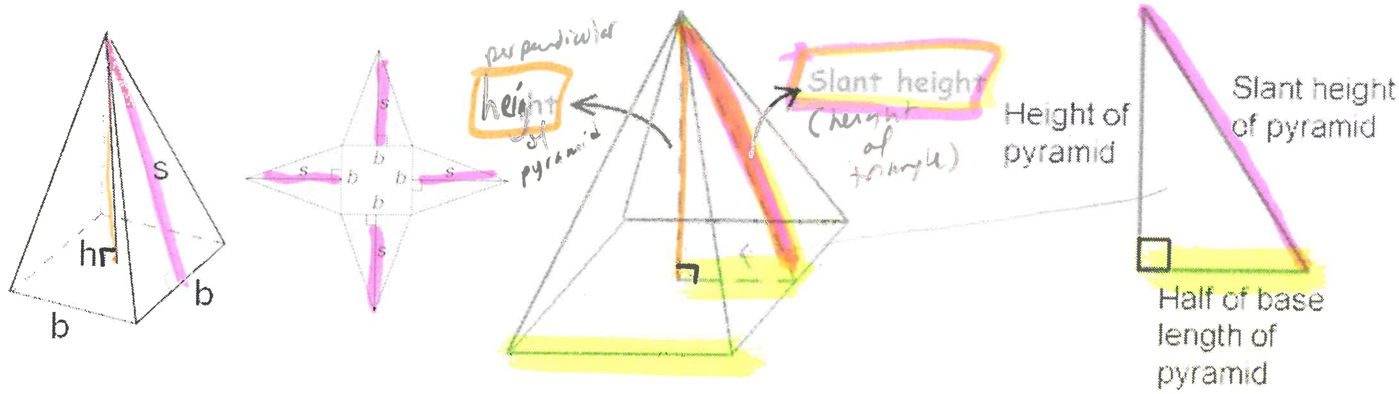


- When the base of a pyramid is a regular polygon, the triangular faces are **congruent** (all the same). All **regular** pyramids have a slant height, which is the height of a lateral face. The slant height of the regular right pyramid is the height of a triangular face (a lateral face).





- The slant height of a right pyramid is the hypotenuse of the right triangle formed by the height and half the base length. (***r*** is half the length of the base).

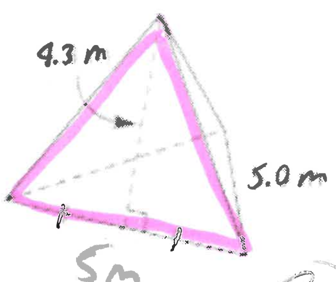


- The Surface area of a right pyramid is the sum of the areas of the triangular faces and the base.

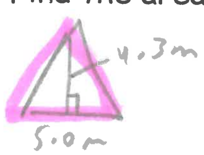
  Square base 4 equal Δ
  rectangle base 2 pair equal Δ

- **Example 1**

Calculate the surface area of this regular tetrahedron to the nearest square meter.



Find the area of one lateral face (one equilateral triangle). (area of triangle: $\frac{bh}{2}$)



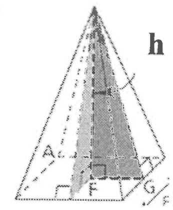
$$A = \frac{bh}{2} = \frac{(5.0)(4.3)}{2} = \frac{21.5}{2} = 10.75 m^2$$

A tetrahedron is made up of 4 equilateral triangles. Take the area of one triangle and multiply by 4 to get the surface area of this tetrahedron.

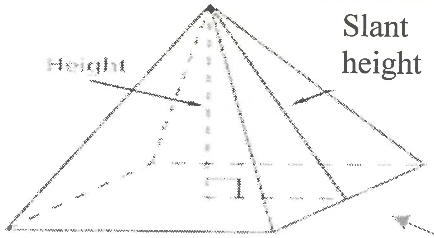
one triangle area = $10.75 \, m^2$
tetrahedron surface area: $(10.75)(4)$
$= 43 m^2$

Example 2

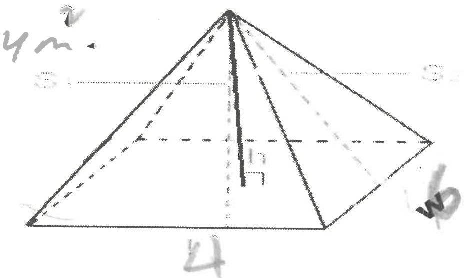
perpendicular pyramid



A **right rectangular pyramid** has base dimensions **4m by 6m, and its height is 8m.** Calculate the surface area of the pyramid to the nearest square metre.



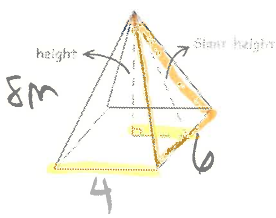
"Right rectangular pyramid" means that the base is a ___rectangle___. It has 4 lateral faces that are 2 pairs of ___congruent___ (equal) triangles.

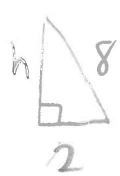


- We can find the rectangle $\boxed{\textbf{base}}$: A = $LW = (4)(6) = 24\,m^2$.
- For the area of the **triangles**, we need the **base** (*which we know*) and the **height** (which we need). The height of the triangle is *the slant height of the pyramid.* The slant height is actually the hypotenuse of the triangle that is formed by the **height** and **half of the base length** of the pyramid.

For the left and right side triangles, visualize the right triangle to find the slant height (height of those triangles).

height = ___8m___ r = $\frac{1}{2}$ length of base ___$\frac{1}{2}(4)=2$___. Find the slant height (hypotenuse)





$h^2 = 2^2 + 8^2$

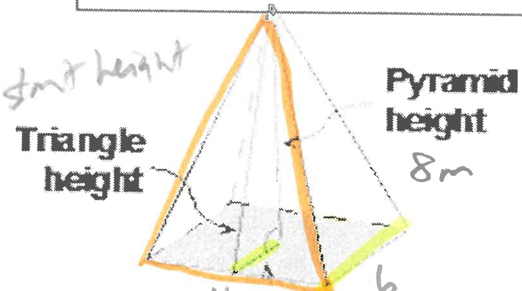
$\sqrt{h^2} = \sqrt{68}$

$h = 8.2462$ ← slant height of pyramid (height of side triangle)

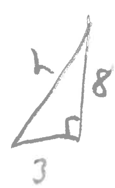
Now find the **area** of **one side triangle**.

$A = \frac{bh}{2} = \frac{(6)(8.2462)}{2} = 24.7386$ side Δ

Repeat the process for the triangles at the front and back.



$\frac{1}{2}$ of 6 $= 3$



$h^2 = 8^2 + 3^2$

$h^2 = 64 + 9$

$\sqrt{h^2} = \sqrt{73}$

$h = 8.5440$

↑ slant height of front of pyramid
height of front Δ

$A = \frac{bh}{2}$

$= \frac{(4)(8.5440)}{2}$

front Δ $A = 17.0880$

Total area of this pyramid = 2 triangles + two triangles + base

$SA = 2(24.7386) + 2(17.0880) + 24$

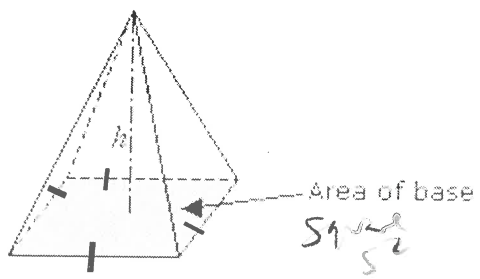
$= 107.6532$

$SA = 108\,m^2$

Formula: Surface area of Right pyramid with Regular polygon base

Instead of finding the sum of all the faces of a right regular pyramid, we can use this formula. (**base** has to have **all sides equal** for this formula). "**s**" is the slant height.

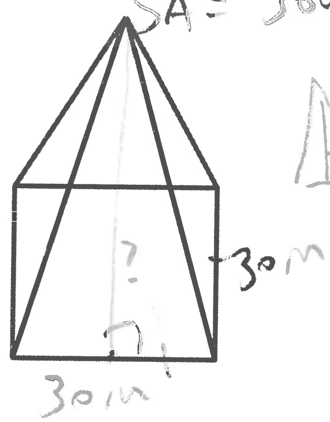
$$\text{Surface area} = \underbrace{\frac{1}{2} s \text{ (perimeter of base)}}_{\text{lateral area }\triangle} + \underbrace{\text{base area}}_{\text{area of base}}$$



Example 3: The surface area of the lateral triangular faces (not including square base) on a right ~~squared~~ square (base is a square) pyramid is $3000 in^2$. The side length of its base is 30 in. Determine the ~~slant~~ height of the pyramid. to tenth of an inch.

If the base is a **regular** polygon with all sides equal, then **all the triangles** will be congruent (equal) (the same).

$SA = 3000 \, in^2$



$SA = \frac{1}{2} S \text{ (perimeter of base)} + \cancel{\text{base area}}$

$3000 = \frac{1}{2} S \, [4(30)]$

$3000 = \frac{1}{2} S \, (120)$

$\frac{3000}{60} = \frac{60S}{60}$

$50 = S$

slant height = 50 in. (not squared)

h, 50, 15 ($\frac{1}{2}$ of 30)

$h^2 + 15^2 = 50^2$

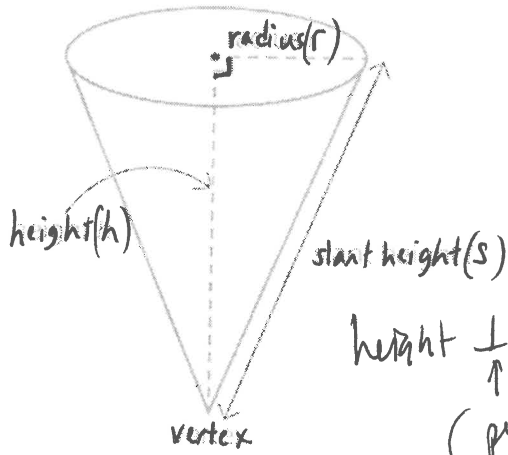
$h^2 + 225 = 2500$
$-225 \quad -225$

$\sqrt{h^2} = \sqrt{2275}$

$h = 47.7 \, in$

Height of pyramid 47.7"

Right Cones



height $\perp$ radius
↑
(perpendicular)

Surface area formula:

area of circle + lateral area

$$SA = \pi r^2 + \pi rs$$

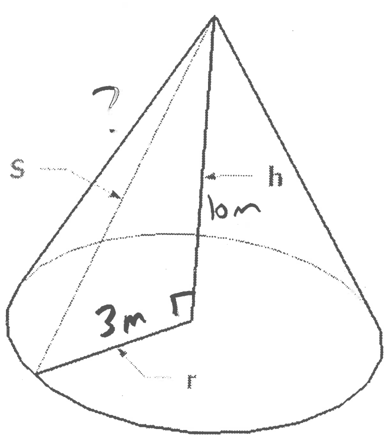
r = radius

s = slant height

Example 1

A right cone has a base radius of 3 m and a height of 10 m. Calculate the surface area of this cone to the nearest square metre.

$SA = \pi r^2 + \pi r s$



To find the surface area, we need the slant height.

The **radius** and height of a cone are **perpendicular**. The slant height forms the hypotenuse of the right triangle that contains the height and radius.

Use Pythagoras.

$s^2 = 3^2 + 10^2$

$s^2 = 9 + 100$

$\sqrt{s^2} = \sqrt{109}$

$s = 10.4403$

$r = 3$

$s = 10.4403$

$SA = \pi r^2 + \pi r s$

$= \pi(3)^2 + \pi(3)(10.4403)$

$= \pi(9) + \pi(31.3209)$

$= 9\pi + 31.3209\pi$

$= 40.3209\pi$ multiply

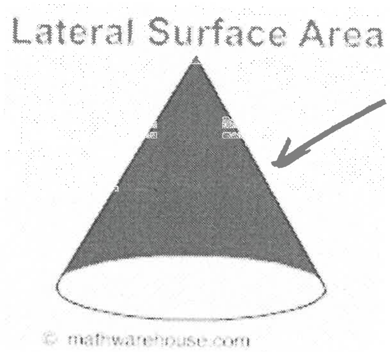
$SA \approx 127\ m^2$

Example 2

not including circle base

r = 5 cm

The lateral surface area of a cone is 220 cm^2. The diameter of the cone is 10 cm. Determine the **height of the cone** to the nearest tenth of a centimetre.



220 cm^2

circle

$$SA = \cancel{\pi r^2} + \pi r s$$

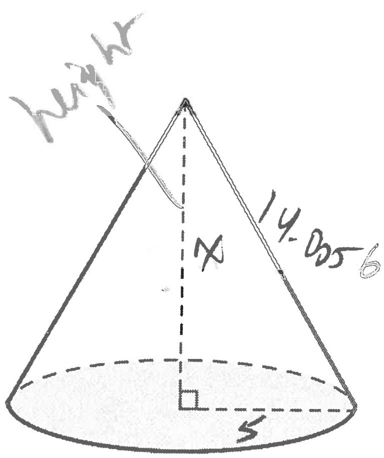
$$S_A = \pi r s$$

$$220 = \pi (5)(s)$$

$$\frac{220}{5\pi} = \frac{(5\pi)s}{5\pi}$$

$$14.0056 \text{ cm} = s$$

$220 \div 5 \div \pi$



$$5^2 + x^2 = 14.0056^2$$

units not squared

$$x = 13.1 \text{ cm}$$

height

Do all at once in calc.

$$\sqrt{(14.0056)^2 - 5^2}$$

Text work: pyramids p. 34 #4,5, 8a, 13 cones #6,7,8b,11

1.5 VOLUME OF RIGHT PYRAMIDS AND RIGHT CONES (p. 36)

Write in COMPLETE SENTENCES.
Use your textbook – pages 36 to 41.

1. What is volume? (*yellow box p. 36*)

   Volume is the amount of space an object occupies.

   It is measured in cubic units.. (units^3)

2. What is **capacity**?

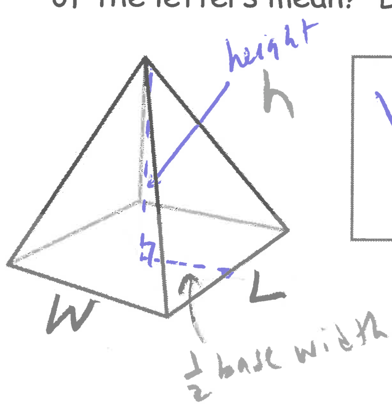
   Capacity is the amount of material a container holds.

It is measured in cubic units or capacity units (like litre, milliliter, ounce, gallon, etc.)

3. Write at least one or two examples of the units we use for volume.

   cm^3
   yd^3
   m^3
   ft^3

4. Write the **formula** for the volume of a **right rectangular pyramid**. What does each of the letters mean? Label this diagram (use a ruler) like on p. 39.



$$V = \frac{1}{3} lwh$$
$$\frac{1}{3} LWH$$

l - length
w - width
h - height (perpendicular height of pyramid)

5. What theorem might will you use to find missing parts of the formula? (p. 38)

Pythagoras-

6. Read the examples on pages 38 and 39. Do you understand them? If not, read them again. Identify parts that confuse you.

p.38 for volume we need lwh.
If given slant height, we use pythagoras.
We use $\frac{1}{2}$ side length of base of pyramid
for base of Δ.
$\text{slant height}^2 = (\frac{1}{2}\text{side length})^2 + (\text{height of pyramid})^2$

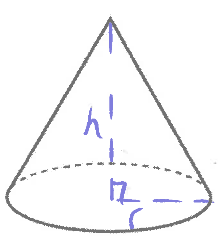
7. Try the "Check Your Understanding" question on page 39. Check your answer.

$V = \frac{1}{3} lwH$

$V = \frac{1}{3}(4.7)(3.6)(6.9)$

$V \approx 38{,}9\ m^3$

8. Write the formula for the volume of a right cone. What does each of the letters mean? Label this diagram (use a ruler) like on p. 40.



r - base radius

h - height (⊥ height of cone)

$V = \frac{1}{3}\pi r^2 h$

9. Read the examples on pages 40 and 41. Do you understand them? If not, read the examples again. Identify parts that confuse you.

diameter = 12 in

radius $= \frac{1}{2}(12) = 6$ in.

10. Try the "Check Your Understanding" question on page 40. Check your answer.

$V = \frac{1}{3}\pi r^2 h$

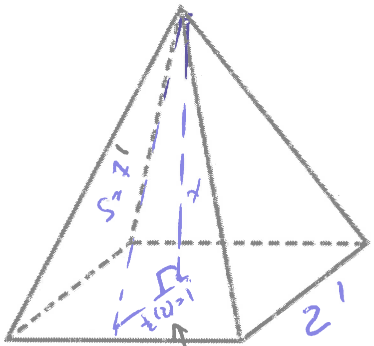
$V = \frac{1}{3}\pi(4)^2(13)$

$V \approx 218 mm^3$

Try it:

Try the "Check Your Understanding" question on page 38 Check your answer.

hyp



Slant height = 7 ft

Square pyramid all sides = 2

x = height

$\frac{1}{2}$ base length $= \frac{1}{2}(2) = 1$

$7^2 = 1^2 + x^2$

$49 = 1 + x^2$

$-1 \quad -1$

$\sqrt{48} = \sqrt{x^2}$

$\sqrt{48} = x$

$6.9282 = x$

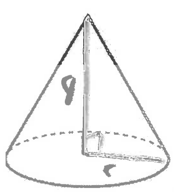
height

$V = \frac{1}{3} lwH$ ← need

$V = \frac{1}{3}(2)(2)(\sqrt{48})$

$V \approx 9 \text{ ft}^3$

Try the "Check Your Understanding" question on page 41 Check your answer.



$V = 300 \text{ m}^3$

$V = \frac{1}{3}\pi r^2 h$

$3(300) = \left(\frac{1}{3}\pi r^2 (8)\right)3$

$\frac{900}{8\pi} = \frac{\pi r^2 (8)}{8\pi}$

$\sqrt{900 \div 8 \div \pi}$

$\sqrt{\frac{900}{8\pi}} = \sqrt{r^2}$

$6 \approx r$

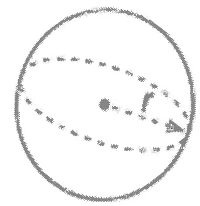
radius ≈ 6 m.

Textbook PRACTICE: Choose at least 7 of the following questions to try, starting on page 42: 4 to 13. Check your answers with the answers in the back of the book.

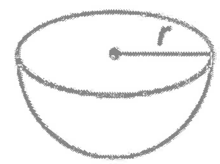
1.6 VOLUME and SURFACE AREA of SPHERES (p. 47)

- **SURFACE AREA OF A SPHERE p. 45**

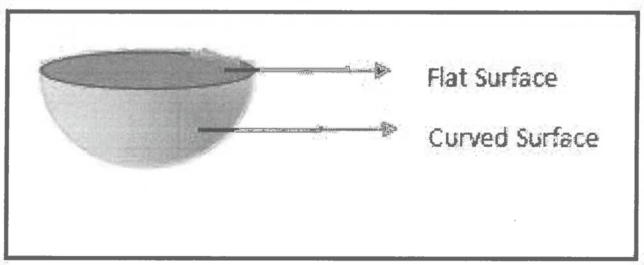
To find the surface area of a sphere, use this formula:
$SA = 4\pi r^2$



To find the surface area of a hemisphere, use this formula:
$SA = 3\pi r^2$

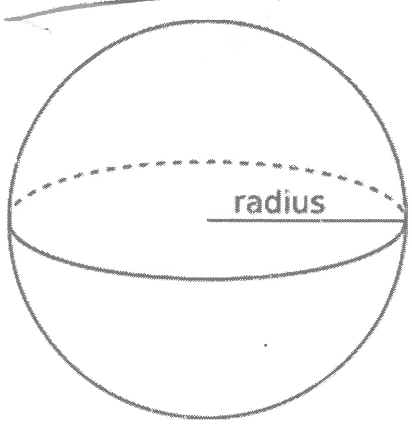


(Hemisphere is half of a **sphere**. It is half the surface area of curved surface of the sphere, PLUS the circle that is the flat surface of the hemisphere. $\therefore SA = \frac{1}{2}(4\pi r^2) + \pi r^2 = 2\pi r^2 + \pi r^2 = 3\pi r^2$)



Example 1

A glass sphere has radius 25 cm. What is the surface area of the sphere, to the nearest square centimetre?



$SA = 4\pi r^2$
$= 4\pi(25)^2$
$SA = 7854 \text{ cm}^2$

Example 2

A globe has surface area 2735 cm^2. Find the radius of the globe, to the nearest tenth of a centimetre.

$$SA = 4\pi r^2$$

$$\frac{2735}{4\pi} = \frac{4\pi r^2}{4\pi}$$

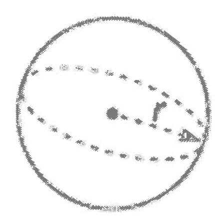
$$\sqrt{\frac{2735}{4\pi}} = r$$

$$15\text{ cm} = r$$

- **VOLUME OF A SPHERE p. 49**

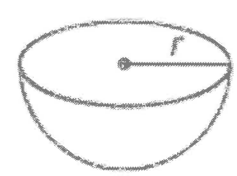
To find the volume of a sphere, use this formula:

$$V = \frac{4}{3}\pi r^3$$



To find the volume of a hemisphere, use this formula:

$$V = \frac{2}{3}\pi r^3$$



(For volume of hemisphere, we simply divide the volune of sphere in half.)

Example 3

A sphere has diameter 8 yd. What is the **volume of the sphere,** to the nearest cubic yard?

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi (8)^3$

$V = 2145 \text{ cm}^3$

$4 \times \pi \times 8^3 = \div 3 =$

Example 4

A hemisphere has radius 6.0 cm.

a) What is the **surface area** of the hemisphere to the nearest tenth of a square centimetre?

$SA = 3\pi r^2$

$SA = 3\pi (6)^2$

$SA = 339{,}2 \text{ cm}^2$

b) What is the **volume** of the hemisphere to the nearest tenth of a cubic centimetre?

$V = \frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi (6)^3$

$V = 452{,}4 \text{ cm}^3$

Textbook work: p. 51 #3c, 4c, 5, 8, 9, 10, 11